

Helicopter Vibration Reduction and Damping Enhancement Using Individual Blade Control

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A control law is presented for a BO-105 helicopter to reduce vibration and to increase damping by the use of individual blade control. H_∞ control synthesis is used to develop a robust controller usable in different operating conditions with different helicopter flight speeds. In simulation, hub load vibration can be canceled (–99%) in three outputs simultaneously, for example in all three hub forces. A reduction in hub vibration, however, does not necessarily lead to reduced vibration in the cabin. Therefore, a finite element model of the flexible fuselage is coupled with the aeromechanical rotor model. The resulting coupled rotor–fuselage model allows vibration to be calculated and controlled at locations in the cabin. A simultaneous vibration reduction of –89% is achieved at the pilot and copilot seats. To reduce gust sensitivity, lag damping must be enhanced, which requires the lag rate of the blades to be fed back. However, the control law is developed with the constraint of no sensors and, consequently, no measurements in the rotating blades. The use of a model-based control strategy enables lag damping to be enhanced from 0.5 to 2–3% critical damping by feedback of the observed lag rates, requiring only measurements of the hub loads.

Nomenclature

A, B, C, D	= state-space matrices
a	= acceleration (with subscript)
d	= disturbance at plant output
d'	= disturbance at plant input
F	= force (with subscript)
F_∞	= state feedback matrix
G	= plant
K	= controller
L_∞	= H_∞ control matrix
M	= moment (with subscript)
N	= number of rotor blades
r	= reference signal
S_y	= output sensitivity
T	= period
t	= time
u	= input vector
u'	= input vector with disturbance
v	= control deviation
W	= weighting function (with subscript)
w	= exogenous input
X_∞	= H_∞ control matrix
x	= state vector
y	= output vector
Z_∞	= H_∞ control matrix
z	= controlled output
γ	= H_∞ norm of augmented closed-loop system
ζ	= critical damping
θ	= blade pitch angle

Subscripts

K	= controller
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x, y, z	= x, y, z axis
$(1), \dots, (N)$	= blade degrees of freedom

I. Introduction

HELICOPTERS suffer from high vibration. This vibration is mostly caused by the main rotor, which operates in a complex aerodynamic flowfield. Changing aerodynamic effects, pilot inputs via the swashplate, and interactions of blades and vortices of preceding blades are causes of oscillations in the flexible rotor blades that are transmitted through the rotor hub and cause vibration in the fuselage. Fuselage vibration leads to human discomfort and fatigue damage of structural components, both of which give reason to aim for helicopter vibration reduction.

Concepts for active vibration control are higher harmonic control (HHC) and individual blade control (IBC). Both methods aim at modifying existing and/or inducing additional forces and moments at the rotor that are opposite in phase and equal in amplitude with the original forces and moments, leading to destructive interference. The original vibration is consequently reduced and ideally canceled out. By IBC, the blades are individually controlled in the rotating frame above the swashplate. In the concept of individual blade root control, the lift of the blade is varied by changes in the pitch of the blade at its root. Therefore, the pitch link rods are substituted by hydraulic actuators, allowing a blade pitch control to be superimposed to the swashplate commands. A four-blade BO-105 helicopter equipped with an individual blade root control system is considered here.¹

One advantage of individual blade root control over other IBC concepts via flaps, twist, etc., is that no changes to the blade are necessary. Thus, the blades do not need to be recertified. However, if the original blades are used, noblade sensors are available and, consequently, no measurements are available in the rotating frame. The availability of only hub load (and possibly fuselage) sensors imposes certain restrictions on the design of the control law, which this paper seeks to examine.

Helicopter vibration reduction belongs to the class of vibration control problems for plants with periodic coefficients. Here, periodicity is a result of the mechanics of the system and cannot be avoided. A linear time-periodic system responds to a sinusoidal input not only with a sinusoid at the excitation frequency, as linear time-constant systems do, but also at additional harmonic frequencies that are spaced by multiples of the plant-periodic frequency.² The introduction of multiblade coordinates³ (MBC) opens up the possibility of the use of a wide range of time-invariant controller

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synthesis techniques because the periodicity of the plant is (partly) preserved in a time-constant MBC representation.

The lightly damped lag motion of the rotor blade is susceptible to various aeroelastic and aeromechanical instabilities. This is the reason why most rotors have mechanical lag dampers that provide additional damping to suppress the occurrence of such aeromechanical phenomena.⁴ Ham et al.⁵ describe how IBC can be used to increase damping actively. However, if the advantage of individual blade root control, namely, the usability of unchanged blades, is to be exploited, control strategies without lag rate sensing are required. One of the main contributions of this paper is the development of a robust observer-based controller to increase lag damping without blade sensors. Preliminary results of this research have been published in Ref. 6.

Friedmann and Millott⁷ give a comparison of various approaches for vibration reduction in rotorcraft by the use of active control. Linear quadratic control is used in Ref. 8 to control six hub forces and moments, and in Ref. 9 to control the hub force in the thrust direction by the use of collective pitch only. Optimal output feedback control strategies are presented in Ref. 10. Aspects of periodic system control are presented in Ref. 11. Active control of fuselage vibration is described in Refs. 12 and 13. Methods for helicopter simulation are presented in Refs. 14 and 15.

An elementary issue in helicopter vibration control is the selection of outputs to be controlled. For a four-blade rotor, typically three hub forces/moments are chosen, for example, the force vector or a combination of the force in thrust direction and the roll and pitch moment.¹⁰ However, a reduction of vibration in selected hub loads can lead to reduced or increased vibration in the remaining hub loads, depending on the dynamic properties of the system. However, vibration is typically increased in outputs not considered for vibration reduction, which is obviously counterproductive to the aim of reduction of vibration that is transmitted to the fuselage.

The final goal of helicopter vibration reduction is to reduce vibration, not necessarily at the rotor hub, but at specific points in the fuselage, for example, at the pilot and copilot seats and in the load compartment. IBC inputs aimed at the reduction of hub loads may not necessarily lead to a simultaneous reduction in the accelerations at specific locations in a flexible fuselage. In Ref. 7, an increase in fuselage acceleration by a factor of two to five from its baseline value was observed when higher harmonic control inputs aimed at minimization of hub shears were introduced. This shows that the model on which the controller design is based is required to include both rotor dynamics and fuselage dynamics. To cope with this, a finite element model of the BO-105 helicopter is integrated with the aeromechanical model of the rotor to develop controllers aimed at minimization of vibration at various locations in the fuselage.

The paper is organized as follows: First, the models used for analysis and design are described, followed by an explanation of the control law design. Next, vibration reduction results are presented, and lag damping enhancement is discussed. Finally, some conclusions are presented.

II. BO-105 Camrad II Rotor Model

Here, a four-bladed BO105 helicopter equipped with an individual blade root control system is considered.¹ The actuation system consists of a conventional swashplate with the pitch rod links substituted by hydraulic actuators. The primary flight control works traditionally via swashplate. The secondary antivibration control is superimposed via the hydraulic actuators in the rotating frame (Fig. 1). Hydraulic power is transported to the rotating frame via a hydraulic slip ring. The effect of IBC on rotor power is difficult to measure because it is only a small percentage of baseline rotor power. Friedmann and Millott⁷ calculate control power required for IBC to be approximately 3–4% of rotor power, which might serve as an upper-bound estimate. In contrast to that, other references suggest IBC as a means to *reduce* rotor power.^{1,16,17}

The aeromechanical analysis software Camrad II¹⁸ is used to derive a state-space model of a BO-105 helicopter rotor with four flexible blades.¹⁰ The rotor blades are modeled as beams by finite elements, which allows flap (out-of-plane), lag (in-plane), and torsion

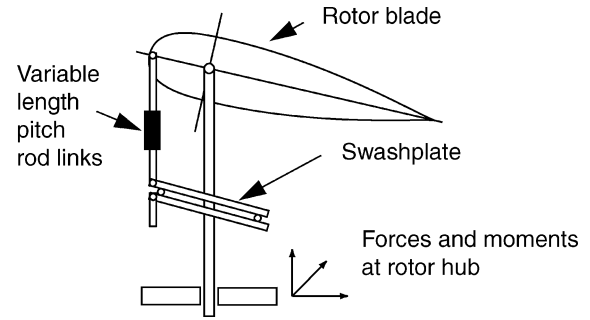


Fig. 1 Individual blade root control.

motion. Aerodynamics are calculated by the use of blade element theory, combined with experimental airfoil tables, a wake model, and uniform inflow theory. The structural blade modes considered are four flap modes, two lag modes, and the first torsion mode. Eight finite elements per blade were used to model the rotor. The highest frequency considered here is the fourth flap mode with 7.97/rev. This was found to be sufficient to predict results for frequencies of up to around 10/rev. Higher-frequency structural and aerodynamical modes, as well as flight mechanical modes, are neglected. Aerodynamic effects of the fuselage and stabilizer are considered by lookup table data. The tail rotor is treated as a rigid rotor without degrees of freedom. The actuators are modeled with first-order dynamics.

The resulting model is linearized at specific operating conditions and can be either linear constant or linear time periodic. The IBC inputs u in the model represent the hydraulic actuator commands. Outputs y are the forces in all three directions and the roll and pitch moments at the rotor hub caused by IBC inputs. The yaw moment is usually not considered in helicopter vibration control.¹⁹ The model includes two elastic blade lag modes, four flap modes, and one torsional mode, for a total of 56 first-order states x . The forces and moments at the hub due to flight, called baseline vibrations, are modeled as output disturbance d . Because of a mechanical filtering effect,²⁰ these baseline vibrations occur at frequencies of iN/rev , with i as integer and N as the number of blades. Vibrations with $i = 1$ are most dominant and solely considered here.

The state and input vectors in the model equation

$$\dot{x} = A(t)x + B(t)u, \quad t \in [0, T]$$

$$y = C(t)x + D(t)u + d$$

are given in MBC that result from a linear Fourier coordinate transformation from single-blade coordinates (SBC).³ This coordinate transformation partly retains the periodicity of the system when time-constant models are used.²⁰

III. BO-105 Fuselage Model

This section presents the fuselage model used to calculate and control vibration at locations in the cabin, such as at the pilot and copilot seats and in the load compartment. An existing finite element model of the fuselage²¹ is used to obtain mode shapes of the flexible structure. The following summary outlines how the mode shapes are implemented into the model in Camrad II and result in a coupled rotor–fuselage model. (See Ref. 22 for a detailed description.) The rotor and fuselage are coupled via pitch/mast bending.¹⁸ This is implemented by specification of the mode shapes (both linear and angular) at the hub. The contribution of the swashplate to coupling is neglected. The fuselage model was developed with the finite element software NASTRAN.²³ The fuselage model consists of 812 points with three translatory and three rotatory degrees of freedom, for a total number of 4872 degrees of freedom.

Based on an analysis²² of the transfer function from IBC inputs to accelerations in the fuselage, there are no significant gain/phase changes from consideration of more than 17 modes. Therefore, 17 fuselage modes are used for the coupled rotor–fuselage model. The highest fuselage mode frequency considered here is 5.68/rev.

The final goal of helicopter vibration reduction is to reduce vibration, not necessarily at the rotor hub, but at specific points in the fuselage, for example, at the pilot seat or in the load compartment. The model, on which the controller design is based, is then required to include both rotor dynamics and fuselage dynamics. (A complete cancellation of all six forces and moments at the rotor hub would be ideal, because no vibration would then be transmitted to the fuselage. However, it is not possible to cancel out completely six outputs due to the limited number of degrees of freedom.²⁷)

A reduction in vibration at one specific location in the fuselage can lead to reduced or increased vibration at other locations, depending on the dynamic properties of the system. Depending on the mission definition of the helicopter, different design objectives are conceivable: For surveillance and transport helicopters, vibration reduction at the pilot/copilot position may be of prime importance, whereas for a rescue helicopter, the load compartment, which is the location of the injured person, becomes more important.

Two different cases are examined: First, vibration reduction is examined at the pilot position only. (For the copilot position, measurements of the vibration level were only available in the y direction for instrumentation reasons.) Second, simultaneous vibration reduction at both the pilot and load compartment positions is considered.

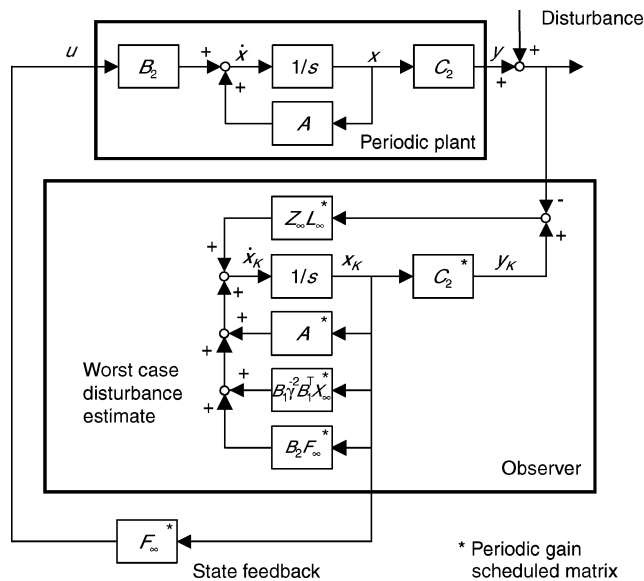


Fig. 3 Observer-based realization of the controller.

In all cases, during the controller design, both off-design locations in the fuselage and at the rotor hub have to be monitored closely to avoid violations of structural load limits.

The results of the simulation given in Fig. 5 show that it is possible to reduce vibration considerably at the pilot position in all three directions (accelerations in the x , y , and z directions). Given the only available measurement at the copilot seat position in the y direction, a simultaneous vibration reduction of -89% at the pilot and copilot position is achieved. Simultaneously, vibration at the load compartment is increased by $+32\%$.

Figure 6 shows the results of the simulation for simultaneous vibration reduction at the pilot seat and in the load compartment. Vibration is reduced in all six outputs, as well as at the copilot position. As expected, because more outputs are considered now than degrees of freedom are available, performance is degraded in comparison to the results for a single location. The average vibration reduction achieved simultaneously at the pilot and copilot seat, as well as in the load compartment, is -47% . Although some fine tuning and adjustment of the weighting functions for the individual outputs might help to distribute the vibration reduction more equally among the outputs, the key finding remains the same: If more independent outputs are considered for vibration reduction than degrees of freedom are available, vibration cannot be canceled, but only reduced moderately.

The result of increased hub loads agrees with those obtained in previous studies. The average increase in hub vibration is $+293\%$. Similar findings have been obtained in Ref. 7, where vertical hub shears increased by a factor of three to six when HHC inputs aimed at minimizing fuselage accelerations were introduced. This indicates that vibration reduction in the helicopter fuselage does not necessarily lead to reduced vibration at the rotor hub and vice versa: That is, a vibration reduction at the rotor hub does not necessarily lead to reduced vibration in the fuselage.

VI. Lag Damping Enhancement

An important issue in the control design is the sensitivity of the closed-loop system to disturbances, for example, gusts affecting one or several rotor blades. The control strategy to increase damping is to use an observer-based controller and to feed back the (observed) rates of the modes to be controlled. The physical mechanism of lag damping augmentation is as follows: The observed lag velocity is fed back to the individual blade pitch control. A blade flapping velocity is, thus, generated, which results in an in-plane moment due to the Coriolis force opposing the lag motion. Because the opposing Coriolis force is proportional to the lag velocity, blade damping is augmented.⁵

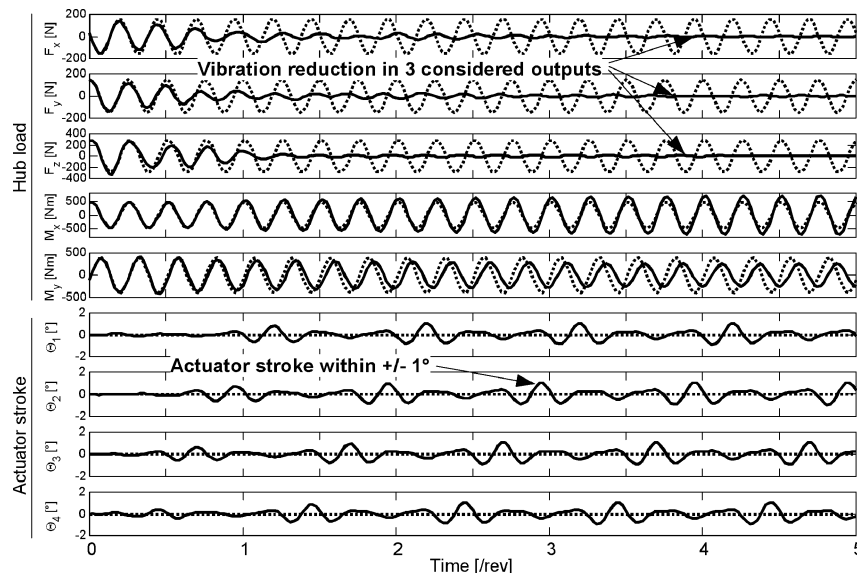


Fig. 4 Vibration reduction in hub force vector: ---, baseline vibration vs —, controlled vibration.

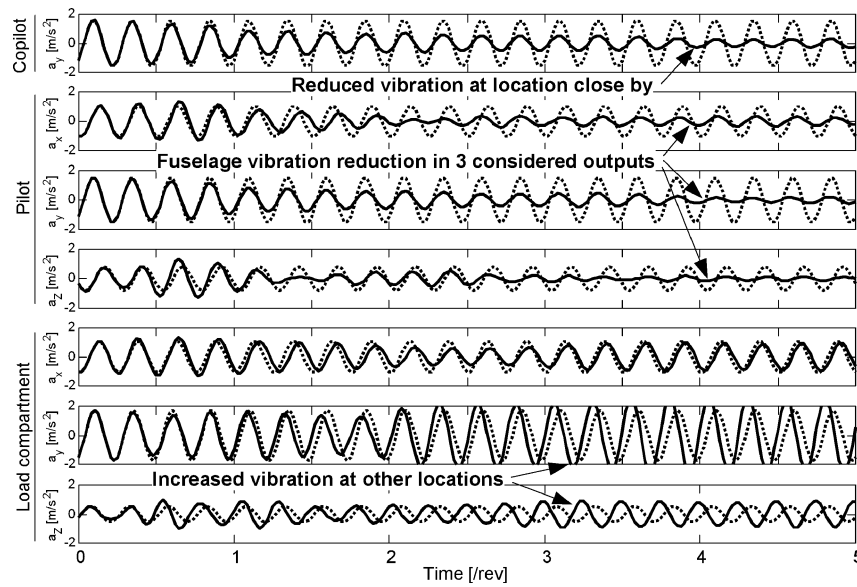


Fig. 5 Fuselage vibration reduction at pilot position: ----, baseline vs —, controlled.

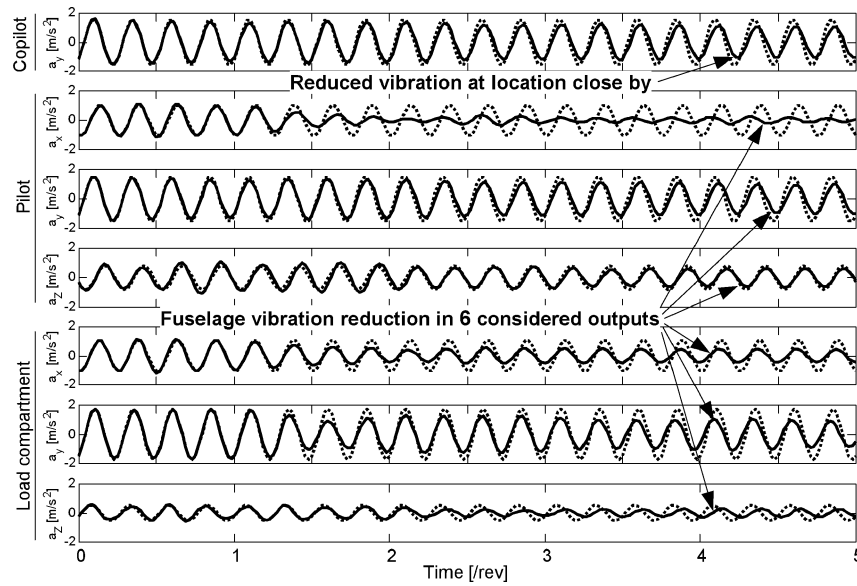


Fig. 6 Simultaneous vibration reduction pilot position and load compartment: ----, baseline vs —, controlled.

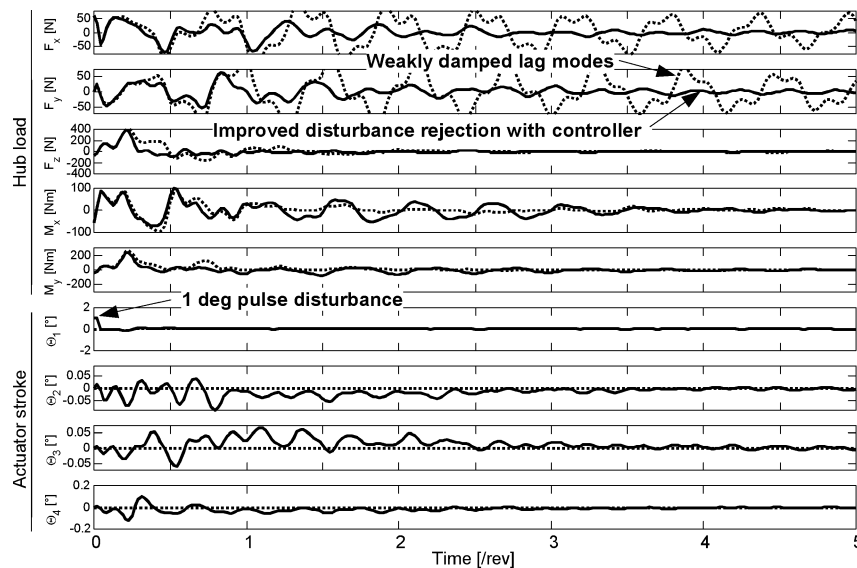


Fig. 7 Disturbance rejection with controller: ----, uncontrolled response vs —, controlled response.

To simulate a gust maneuver, one rotor blade is pitched up one degree for the duration it takes the blade to advance 10 deg, that is, a pulse input to one blade is simulated. The results are compared for the uncontrolled and the controlled cases. Here, the forces in the x and y direction are most interesting because the weakly damped lag modes mostly affect F_x and F_y . Figure 7 shows the improvements achieved by the controller with respect to decay times.

By definition, the observer requires that the modes to be controlled are observable in the measured outputs. A frequency domain analysis of the plant with respect to controllability and observability revealed that the cyclic forms of the lag modes are not expected to cause any problems, whereas the collective lag form is only observable in M_z . Moreover, the differential lag mode cannot be controlled by a time-constant controller based on a time-constant approximation of the time-periodic plant because the differential

mode is not observable ("reactionless" mode) in the time-constant system.⁶

To analyze the influence of the availability of M_z as a measurement, two time-constant controllers are designed for the plant. Both controllers use all three forces and the moments about the pitch and yaw axes, whereas the second controller is also able to utilize a measurement of the moment about the torque axis M_z . The section of the pole map containing the most relevant second lag mode is given in Fig. 8. N roots are shown. The locations of the collective and differential roots correspond to the natural frequency and damping of the mode, whereas the imaginary parts of the roots of the cyclic modes are shifted by $\pm n/\text{rev}$ in MBC. Both controllers increase damping in the cyclic forms. With both controllers, the critical damping of the cyclic modes is increased from the minimum 0.5% open-loop damping to 2% in the closed loop. However, collective lag mode

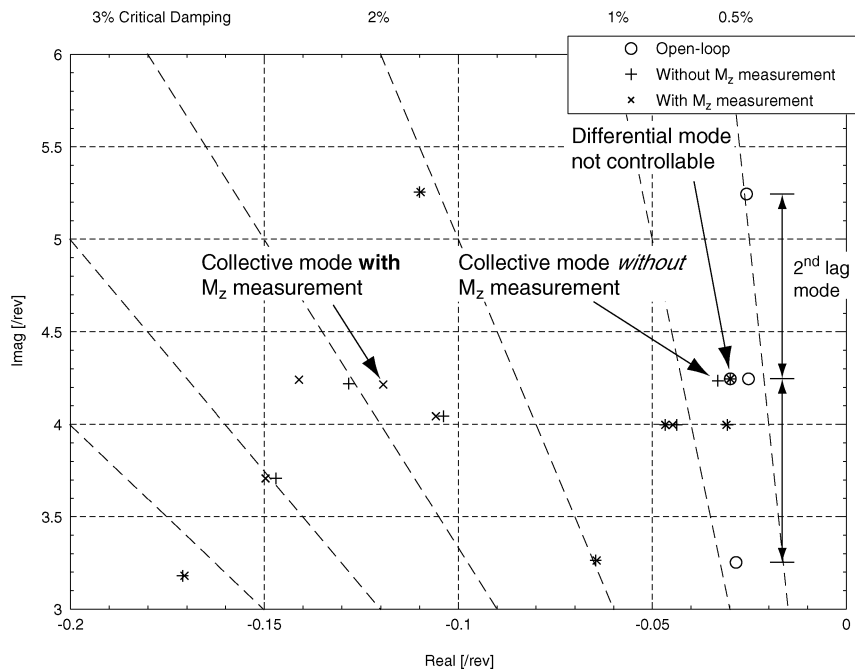


Fig. 8 Open-loop pole locations vs closed-loop pole locations, comparison of controller with and without measurement of M_z available for feedback.

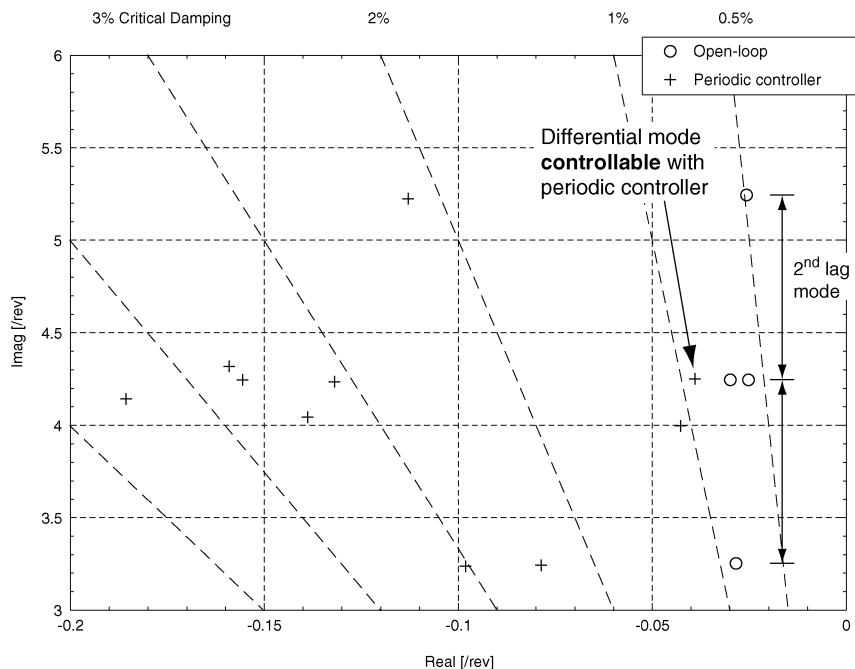


Fig. 9 Open-loop pole locations vs Poincaré exponents (closed-loop pole locations of the Floquet-transformed) periodic closed-loop system with time-periodic controller (with M_z feedback).

damping can only be increased to $> 2\%$ when a measurement of M_z is available, whereas the collective pole remains almost unchanged when M_z cannot be fed back. In both designs, the differential mode remains unchanged.

Figure 9 shows the results for a time-periodic controller. The controller is based on the preceding design parameters, including feedback of M_z , with the exception of time periodicity. The open-loop pole locations of the plant are compared with the closed-loop system Poincaré exponents (closed-loop pole locations of the Floquet-transformed time-periodic closed-loop system). Although the results of the cyclic and collective modes are only slightly improved, the key result is that the time-periodic controller allows one to control the differential mode. Differential second lag mode damping is increased to approximately 1% critical damping. The possible differential damping enhancement is not as large as for the cyclic or collective form. This is due to the dynamic properties of the plant and not to periodicity or the gain-scheduled time-periodic implementation of the controller because time-constant controller designs for fictitious time-constant rotors "fixed" at specific azimuthal positions yield similar results. Although the differential damping enhancement is smaller than that for the cyclic and collective forms, the time-periodic controller allows one to increase second lag mode damping from minimal 0.5% critical damping to 1% in the differential form and to 2–3% in the collective and cyclic form. With a controller design based on the constant coefficient approximation, it is not possible to control the differential form from the nonrotating system.

VII. Conclusions

A control law to reduce vibration and increase lag damping in helicopters via IBC has been developed. H_∞ control synthesis was used to design one robust controller based on a reduced-order model that can be used to alleviate vibration in different operating conditions at different helicopter flight speeds. The damping enhancement achieved leads to a significantly reduced gust sensitivity.

With regard to hub loads, vibration can be canceled (–99%) in three outputs simultaneously, for example, all three hub forces. By the use of the same controller as designed for high-speed flight at a different operating point with low-flight speed, oscillatory loads were also reduced by –96%, demonstrating the robustness of the controller.

As in previous studies, a reduction in hub vibration does not necessarily lead to reduced vibration in the cabin. Fuselage accelerations increased by a factor of up to three when IBC inputs aimed at minimizing hub loads were introduced. Therefore, a finite element model of the flexible fuselage was coupled with the aeromechanical rotor model. The resulting coupled rotor–fuselage model was used to calculate and control vibration at locations in the cabin, such as at the pilot and copilot seats and in the load compartment. With the focus of cabin accelerations, simultaneous vibration reduction at the pilot and copilot seats of –89% was achieved, at the expense of an increased level of vibration of +32% in the load compartment.

The use of a model-based control strategy enabled lag damping to be enhanced from 0.5 to 2–3% critical damping without the use of dedicated lag rate sensors in the blades. Only the hub loads must be measured. However, because the lag rate of the blades in the rotating frame has to be reconstructed from measurements in the nonrotating system, some restrictions apply. To increase damping in the cyclic modes is straightforward, whereas damping enhancement in the collective mode requires measurement of the torque moment. Finally, differential lag mode damping can only be increased via periodic control.

Flight tests of the proposed control law are planned on the BO 105 helicopter equipped with the individual blade root control system.

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